Even/Odd Functions (1.5) + Linear Approximation Homework

1. Let $g(x)$ be an odd function. If $g(x)$ is decreasing and concave down on the interval $(-4,-2)$, then what can be said of $g(x)$ on the interval $(2,4)$? Select one of the following.
   A) $g(x)$ is increasing and concave up on the interval $(2,4)$
   B) $g(x)$ is decreasing and concave up on the interval $(2,4)$
   C) $g(x)$ is increasing and concave down on the interval $(2,4)$
   D) $g(x)$ is decreasing and concave down on the interval $(2,4)$

2. Which one of the following could be a table of values of an odd cubic function?

(A)

x	-3	-2	-1	0	1	3
$f(x)$	-9	-1	1	2	-1	9

(B)

x	-3	-2	-1	0	1	3
$f(x)$	-18	3	6	0	-6	18

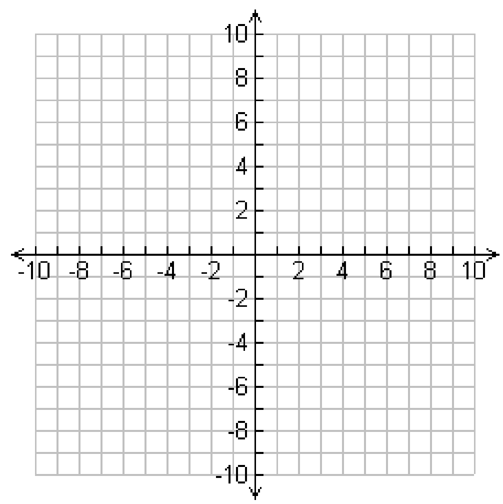
(C)

x	-3	-2	-1	0	1	3
$f(x)$	9	-4	7	0	-7	-9

(D)

x	-3	-2	-1	0	1	3
$f(x)$	5	4	-1	0	1	5

3. On the graph provided to the right, sketch a polynomial function $f(x)$ for which the following are true:
   - $f(x)$ is an even function
   - $f(-6) = 0$
   - $f(x)$ is decreasing and the rate of change of $f(x)$ is decreasing on $(0,3)$
   - $f(x)$ is decreasing and the rate of change of $f(x)$ is increasing on $(3,5)$
   - $f(x)$ is increasing and the rate of change of $f(x)$ is increasing on $(5,\infty)$



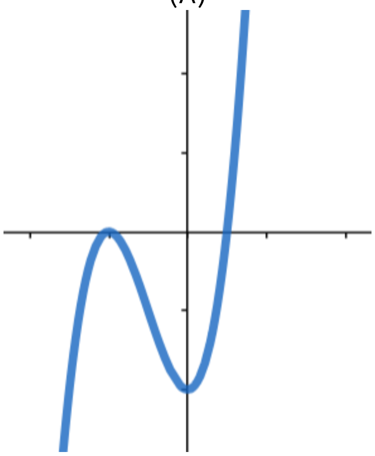
Each of the following questions describes a polynomial function. However, each scenario is impossible. Give a reason why the proposed polynomial is impossible.

4. $\lim_{x\to-\infty} f(x) = -\infty$, $\lim_{x\to\infty} f(x) = \infty$, 1 is a zero of f with multiplicity 2, f is an even function.
5. The average rate of change of $f(x)$ over the interval $[-4,4]$ is $\frac{1}{2}$, $f(x)$ has a global maximum, $f(x)$ does not have a global minimum, $f(x)$ is an even function
6. The leading coefficient of f is negative, the graph of f is tangent to the x-axis at $x = -3$, f is an odd function, f is cubic

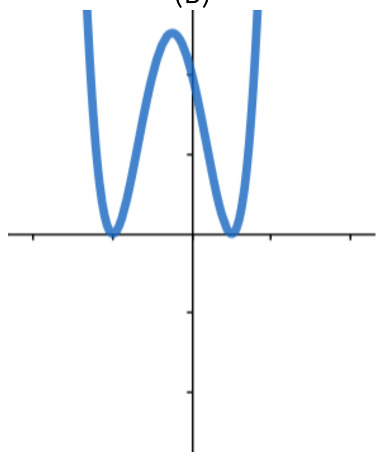
Below are nine descriptions of polynomials and nine graphs. Match each description to the letter of the corresponding graph. Use each graph/description exactly once. There are no repeats.

7. An odd function with three real zeros, all of multiplicity 1
8. The graph of $y = (x-1)(x+2)^2$
9. An even, quartic function (quartic meaning degree=4)
10. The graph of $y = (x-1)(x^2+x+2)$
11. Three distinct real zeros and $\lim_{x\to\infty} f(x) = \infty$
12. The graph of $y = (x-1)^2(x+2)^2$
13. An odd cubic function for which (x^2+2) is a factor.
14. A quartic function for which $-1+2i$ is a zero (quartic meaning degree=4)
15. An even function for which x is a factor.

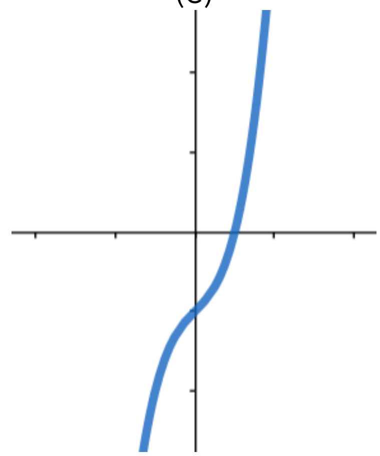
(A)



(B)



(C)



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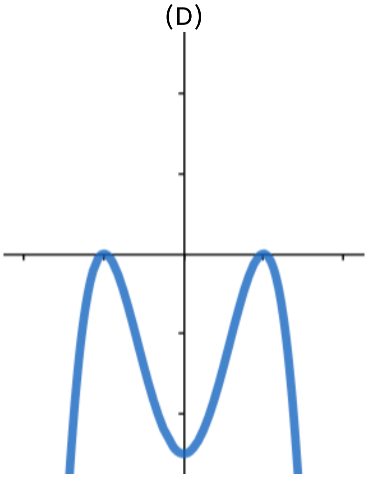
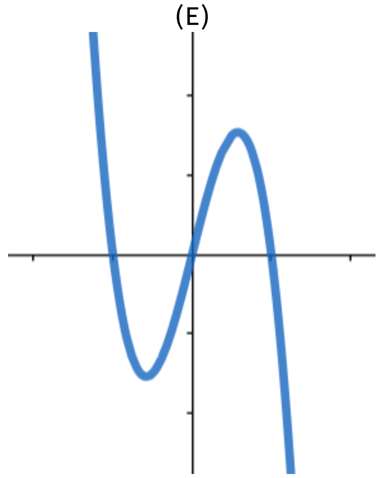
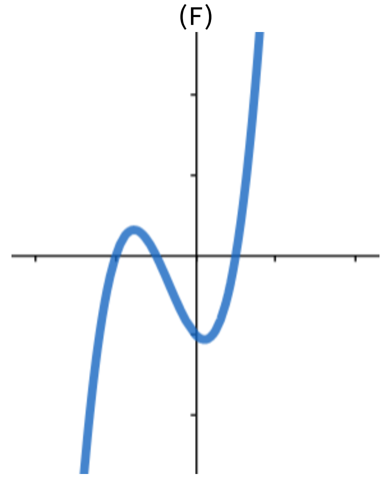


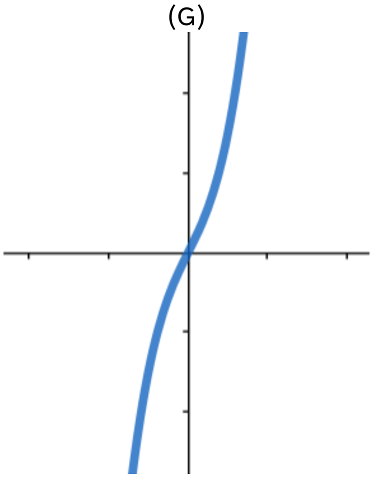
(E)



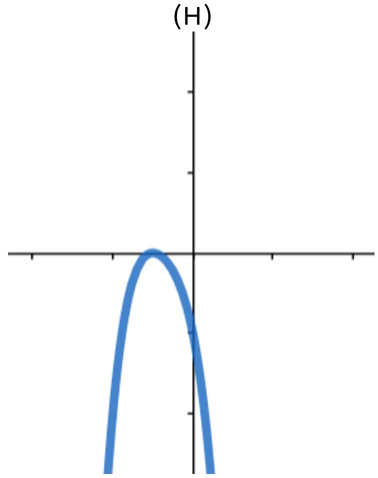
(F)



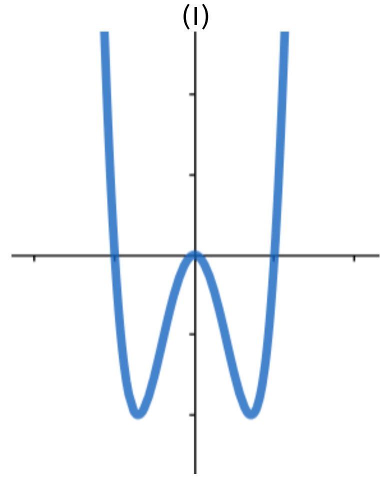
(G)



(H)



(I)



You may use a calculator to aid you in questions #16-18, but there is still work to be shown. For questions asking you to find average rate of change, there should be a quotient in your work that looks like you did change in output value divided by change in input value. For questions that ask you to perform linear approximation, there should be an equation in which you solved for an unknown. For part (iii) of each question, a sketch of a curve with a secant line passing through two labeled points should be included.

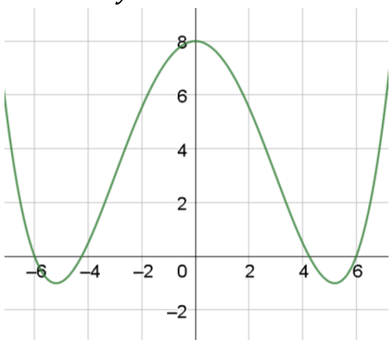
16. Consider the function $R(t) = -t^3 + 5t + 6$
    - (i) Find the average rate of change of $R(t)$ from $t = 1$ to $t = 4$. Express your answer as a decimal approximation. Show the computations that lead to your answer.
    - (ii) Use the average rate of change found in (i) to approximate $R(4.25)$. Show the work that leads to your answer.
    - (iii) The graph of $y = R(t)$ is concave up for $t > 0$. Let $A(t)$ be the approximation of $R(t)$ obtained based on the average rate of change found in (i). For $1 < t < 4$, $A(t)$ overestimates $R(t)$. Explain why this is the case.

17. Consider the function $S(t) = \sqrt{t}$
    - (i) Find the average rate of change of $S(t)$ from $t = 4$ to $t = 16$. Express your answer as a decimal approximation. Show the computations that lead to your answer.
    - (ii) Use the average rate of change found in (i) to approximate $S(12)$. Show the work that leads to your answer.
    - (iii) Do you remember from Algebra II what the square root graph looks like? That's okay, I'll tell you. $S(t)$ is concave down over its entire domain. Explain why, without knowing the actual value of $\sqrt{12}$, we are confident that our answer from (ii) underestimates the actual value of $\sqrt{12}$.

18. Starting in the year 2000, I began carefully tracking the net worth of Scrooge McDuck (Donald Duck's uncle). In 2008 ($t = 8$), his net worth was $\$29.1$ billion. In 2010 ($t = 10$), his net worth was $\$33.5$ billion. Let $W(t)$ be a function that models Mr. McDuck's net worth, in billions, t years after the year 2000.
    - (i) Find the average rate of change of $W(t)$ over the interval from $t = 8$ to $t = 10$. Express your answer as a decimal approximation. Show the computations that lead to your answer.
    - (ii) Use the average rate of change found in (i) to approximate $W(16)$. Show the work that leads to your answer.
    - (iii) If $W(t)$ is always changing at an increasing rate, then is our approximation from part (ii) an overestimate or underestimate of the actual value of $W(16)$? Explain how you know.

Even/Odd Functions (1.5) + Linear Approximation HW Answers

1. B
2. B
3. Answers may vary, but the graph should be decreasing concave down on $(0,3)$, decreasing concave up on $(3,5)$, and increasing concave up on $(5,\infty)$. Because the function must be even, there should be symmetry over the y-axis.



4. An even polynomial must have an even degree and must have left end behavior that matches right end behavior. This contradicts the limits given.
5. If f is an even function, then $f(-4) = f(4)$. But then the average rate of change of $f(x)$ over the interval $[-4,4]$ must be $\frac{f(4)-f(-4)}{(4)-(-4)} = \frac{f(4)-f(4)}{8} = \frac{0}{8} = 0$.
6. If f is odd and its graph is tangent to the x-axis at $x = -3$, then by symmetry its graph must also be tangent to the x-axis at $x = 3$. These two zeros of even multiplicity mean that the degree of f must be at least 4. f cannot be cubic.
7. E
8. A
9. D
10. C
11. F
12. B
13. G
14. H
15. I
16.
    (i) $\frac{R(4)-R(1)}{(4)-(1)} = \frac{(-38)-(10)}{(4)-(1)} = \frac{-48}{3} = -16$

    (ii) $\frac{R(4.25)-R(4)}{(4.25)-(4)} \approx -16$

    $\frac{R(4.25)+38}{0.25} \approx -16$

    $R(4.25) + 38 \approx -4$

    $R(4.25) \approx -42$

    (iii) $A(t)$ is the secant line between $\big(1, R(1)\big)$ and $\big(4, R(4)\big)$. $R(t)$ is concave up for $t > 0$ and is therefore concave up between $t = 1$ and $t = 4$. Since R is concave up, the graph of R is below the graph of A on the interval $1 < t < 4$. Thus, $A(t)$ overestimates $R(t)$ on that interval.
17.
    (i) $\frac{S(16)-S(4)}{(16)-(4)} = \frac{4-2}{16-4} = \frac{2}{12} = \frac{1}{6} = 0.167$

    (You MUST write as a decimal to earn credit since the question specifically asked for it. You MUST have at least three decimal places in AP Precalculus. You are allowed more.)

(ii) $\frac{S(12)-S(4)}{(12)-(4)} \approx \frac{1}{6}$

$\frac{S(12)-2}{12-4} \approx \frac{1}{6}$

$\frac{S(12)-2}{8} \approx \frac{1}{6}$

$S(12) - 2 \approx 1.333333$

$S(12) \approx 3.333$

(iii) We are comparing the output of the concave down function $S(t)$ to the secant line passing through $(4, S(4))$ and $(16, S(16))$. Because S is concave down, it is above the secant line between $t = 4$ and $t = 16$. This includes $t = 12$. So, the approximation from part (ii) underestimate the actual value of $S(12) = \sqrt{12}$.

18.

(i) $\frac{W(10)-W(8)}{10-8} = \frac{33.5-29.1}{10-8} = \frac{4.4}{2} = 2.2$

(ii) $\frac{W(16)-W(10)}{(16)-(10)} \approx 2.2$

$\frac{W(16)-33.5}{6} \approx 2.2$

$W(16) - 33.5 \approx 13.2$

$W(16) \approx 46.7$

(iii) "$W(t)$ is always changing at an increasing rate" means that $W(t)$ is always concave up. We are comparing the outputs of the concave up function W to the outputs of the secant line passing through $(8, W(8))$ and $(10, W(10))$ for $t > 10$. Because W is concave up, the graph of W is above the secant line for $t > 10$. This includes $t = 16$. So, $W(16)$ is greater than the estimate based on the average rate of change. The answer in part (ii) underestimates $W(16)$.